Technical Report

On

Value at Risk (VaR)

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INTRODUCTION:

In this report, my intentions are to analyze the market risk by using Value at Risk (VaR) technique. The main focus of this report would be to describe the different methods which are used in calculation of VaR and apply them on Pakistani market.

ABSTRACT:

Over the last few years, Value at Risk has been universally accepted as a measure of market risk in financial institutions. A lot of research has been done in the field of Value at Risk leading to the development of differing approaches to estimate Value at Risk.

However each method has its own set of assumptions and there is very little consensus on the preferred method to estimate Value at Risk. Since all existing methods involve some tradeoff and simplifications, determining the best methodology for estimating Value at Risk becomes an empirical question for implementing the most suitable model. The challenge of this work is to come up with the best and easily implemental approach suitable to Karachi Stock Exchange data and apply time series models for calculating Value at Risk. The working identifies the path for future research to improve the performance of models. The Value at Risk models are evaluated over the two sample periods. The two periods serve to validate the performance of models over time. The best models (EWMA and GARCH) models were reevaluated for the extended forecast sample period and it was found that GARCH models performed consistently over the time. This makes use of both parametric and non parametric models and also proposes some of the models to estimate Value at Risk. Performance evaluation of the risk metrics, GARCH models and historical simulation Value at Risk models are outlined and assumptions tested on Karachi stock exchange index. Overall the risk metrics model with decay factor of 0.94 performs better than all other models when comparing the accuracy of Value at Risk estimates in first sample period. However over the both forecast sample periods the GARCH models perform consistently.

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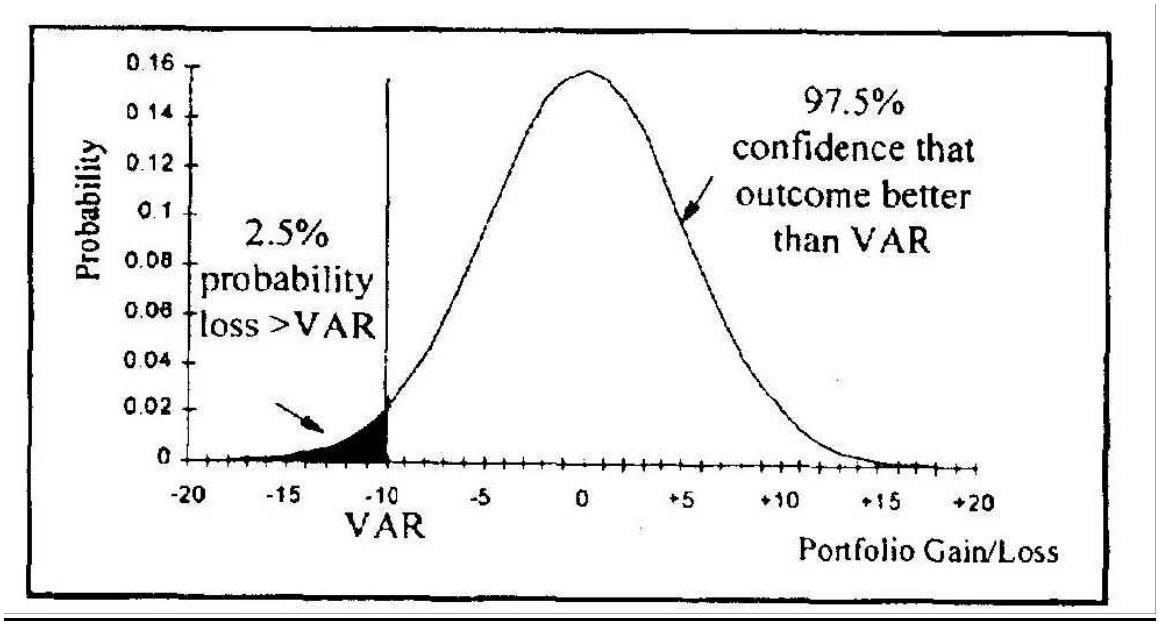
What is VaR, Value at Risk?

What is the most I can lose on this investment?

The VaR is the maximum amount at risk to be lost from an investment (under 'normal' market conditions) over a given holding period, at a particular confidence level. As such, it is the converse of shortfall probability, in that it represents the amount to be lost with a given probability, rather than the probability of a given amount to be lost.

In Stock Market, the Value at risk, or VaR, is a measure- use to estimate how much the value of shares or of a portfolio of shares could decrease over a certain time period (usually over 1 day or 3 days) under daily movement of share prices.

It is used by Stock Exchanges to measure the market risk or volatility risk of the transacted but unsettled shares. Banks and other financial institutions also use VaR based system for their risk management regimes.



The Idea behind VaR:

The most popular and traditional measure of risk is volatility. The main problem with volatility, however, is that it does not care about the direction of an investment's movement: a stock can be volatile because it suddenly jumps higher. Of course, investors are not distressed by gains.

For investors, risk is about the odds of losing money, and VaR is based on that common-sense fact. By assuming investors care about the odds of a really big loss, VAR answers the question, "What is my worst-case scenario?" or "How much could I lose in a really bad month?"

1) Value at Risk can be viewed as a simple indicator of portfolio risk which can be easily communicated to senior management. It monetarily quantifies the exposure of the portfolio to market fluctuations.

2) Value at Risk assumes that future profits and losses can be expressed by observing historical data and projecting it into the future or by making an assumption that the future profits and losses stem from the same distribution.

3) Value at Risk depends upon the length of the holding period to be analyzed. The daily Value at Risk will always be less than the monthly Value at Risk. The choice of the holding period is subjective and depends completely upon the purpose Value at Risk is directly proportional to the confidence level.

4) The higher the confidence level greater the Value at Risk.

The assumptions behind VaR calculations:

There are several statistical assumptions that must be made in order to make VaR calculations tractable. First, we consider the ***stationarity requirement***. That is, a 1 percent fluctuation in returns is equally likely to occur at any point in time. Stationarity is a common assumption in financial, because it simplifies computations considerably.

A related assumption is the ***random walk*** assumption of intertemporal unpredictability. That is, day-to-day fluctuations in returns are independent; thus, a decline in the KSE 100 index on one day of x percent has no predictive power regarding returns on the KSE 100 index on the next day. That is, if the mean daily return is zero, then the best guess estimate of tomorrow's price level (e.g., the level of the KSE 100 index) is today's level. There is no relevant information available at time t that could help forecast prices at time $t + 1$.

Another straightforward assumption is the ***non-negativity requirement***, which stipulates that financial assets with limited liability cannot attain negative values.

The most important assumption is the ***distributional assumption.*** In the simple equity portfolio example, we assumed that daily return fluctuations in the KSE 100 index follow a normal distribution with a mean of zero and a standard deviation of 100 basis point. We should examine the accuracy of each of these four assumptions. First, the assumption of a zero mean is clearly debatable, since at the very least we know that equity prices, in the particular case of the KSE 100, have a positive return.

The most questionable assumption, however, is that of normality because evidence shows that most securities prices are not normally distributed.

Calculation of Risk (parameters):

VAR is typically calculated for one day time period known as the holding period.

1) A 99% confidence level means that there is (on average) a 1% chance of the loss being in excess of that VaR.

2) Value at Risk (VaR) calculates the maximum loss expected (or worst case scenario) on an investment, over a given time period and given a specified degree of confidence.

3) VaR is measuring risk and providing warning signals. VaR is only an estimate. This means there is a likelihood or probability of you losing not more than the VaR on most days.

If you want to be more conservative in knowing your likely losses, you have to increase the confidence level to, say, 99 per cent. Suppose, the daily VaR at a 99 per cent confidence level is Rs 20,000 on the portfolio, it means you are likely to lose not more than Rs 20,000 in 99 out of 100 trading days. Notice that VaR increases as you increase the confidence level.

What VaR does not do?

- VaR does not give a consistent method for measuring risk. Different VaR models will give different VaR figures.
- VaR only measures risks that can be captured through quantitative techniques. It does not measure political risk, personal risk, liquidity risk or regulatory risk & operational risk.

Example:

Consider a trading portfolio. Its market value of investment in Karachi Stock Exchange today is known, but its market value tomorrow is not known. The investment holding that portfolio might report that its portfolio has a 1-day VaR of Rs4 million at the 95% confidence level. This means that in a 'normal' 1-day period, the Stock Exchange believes there is a 95% probability that the change in its portfolio's value will be less than $4 million; conversely, the bank believes there is a 5% probability that the portfolio's value will change by more than Rs4 million. Note that, as the name implies, VaR is a measure of possible losses, so the implication is that, with probability 95%, the bank stands to lose less than $4 million in a given 1-day period.

Value at Risk (VaR) Methodologies:

A Variety of models exist for estimating VaR. Each model has its own set of assumptions but the most common assumption is that historical market data is our best estimator for future changes. Common models include:

There are various methods for computing VaR such as parametric and non parametric models. The simplest of the models is the parametric VaR, which assumes that the daily portfolio returns are ``normally distributed''.

1) **A Single Underlying Market Variable**
2) **Variance / Covariance Method**
3) **Historical Simulation Method.**
4) **Exponentially Weighted Moving Averages (EWMA).**
5) **Monte Carlo Simulation Method.**

Inputs into VaR calculations:

VaR calculations require assumptions about the possible future values of the portfolio some time in the future. There are at least three ways to calculate a rate of return from period t to $t+1$:

- absolute change $\Delta S_{t,t+1} = s_{t+1} - s_t$
- simple return $R_{t,t+1} = (s_{t+1} - s_t)/s_t$ (or $1 + R_{t,t+1} = s_{t+1}/s_t$)
- continuously compounded return $r_{t,t+1} = \ln(s_{t+1}/s_t)$.

1) A Single Underlying Market Variable:

1). Compute the exposure to market risk. Here is Rs x (price) million

2). Evaluate the risk of the position. Estimate the standard deviation of the daily return of equity price is x% (return) and assume the returns are normally distributed, for the chosen confidence level 95%, the VaR percentage (%) is 1.65 * x%

3). The VaR of the position = x million* VaR%

2) Variance Covariance Method:

Variance Covariance Method assuming that risk factor returns are always (jointly) normally distributed and that the change in portfolio value is linearly dependent on all risk factor returns,

It assumed that the returns for all the market Variables are normally distributed.

The Variance covariance approach is based on the assumption that the underlying market factors have a multivariate Normal distribution. Using this assumption (and other assumptions detailed

below), it is possible to determine the distribution of mark-to market portfolio profits and losses, the loss that will be equaled or exceeded *x* percent of the time, i.e. the value at risk.

A portfolio consists of a single instrument, the Karachi stock index with a holding period of one day and the desired a probability is 5%. The distribution of possible profits and losses on this simple portfolio can be represented by the normal probability density function. This distribution has a mean of zero, which has been found to be reasonable because the expected change in portfolio value over a short holding period is almost always close to zero. Given the properties of the normal distribution about 66% of distribution area is between -1 and +1 standard deviations and about 95% of the area is between -2 and + 2 standard deviations.

A standard property of the Normal distribution is that outcomes less than or equal to 1.645 standard deviations below the mean occur only 5 percent of the time.

The possible values of the market Variables can be captured by Variance Covariance Matrix (VCM) of the market returns, which can be computed by SMA or EWMA Models.

Assumptions (Variance Covariance):

(1) The portfolio is composed of assets whose deltas are linear, more exactly: the change in the value of the portfolio is linearly dependent on (i.e. is a linear combination of) all the changes in the values of the assets, so that also the portfolio return is linearly dependent on all the asset returns.

(2) The asset returns are jointly normally distributed.

The implication of (1) and (2) is that the portfolio return is normally distributed because it always holds that a linear combination of jointly normally distributed Variables is itself normally distributed.

Formula:

That is, if a probability of 5 percent is used in determining the value at risk, then the value at risk is equal to 1.65 times the standard deviation of changes in portfolio value. Using this fact, Value at Risk can be calculated as

$$\textbf{Value at Risk} = a * s * P\,(t)^{\wedge -0.5}$$

Where

a is the Z score corresponding to the given probability level

s is the standard deviation of the returns of the asset

P is the market value of the asset or portfolio, and

t is the horizon scale factor.

It is evident that the computation of the standard deviation of changes in portfolio value is at the core of this approach. While the approach is based on just a handful of formulas from statistics textbooks, it captures the determinants of value at risk mentioned above. It identifies the intuitive notions of Variability and co-movement with the statistical concept of standard deviation (or Variance) and correlation. These determine the Variance covariance matrix of the assumed Normal distribution of changes in the market factors.

Advantages:

Widely used in many market and is the basic method for evaluating VaR

Simplest method to understand and implement and is the least computationally demanding

Disadvantages:

The model is not very good in energy market and not suitable for portfolios of options (assumption of normally distributed returns)

The volatilities and correlations are based on past history

Obtain the data of KSE-100 Index on any particular scrip

Obtain the log returns for the period

Fit the Volatility estimate models through Moving Average & Risk metrics® (EWMA), to compute the standard deviation of log returns

Moving Average 50

$$\sigma_t^2 = \frac{1}{m-1}\sum_{j=1}^{m}(y_{t-j} - \hat{\mu}_t^m)^2$$

m= moving window = 50
y_{t-j} is a return series and
μ_t^m is the mean

Risk metrics

$$\sigma_t^2 = \lambda\sigma_{t-1}^2 + (1-\lambda)(y_{t-1} - \hat{\mu}_t)^2,$$

c is exponentially declining factor . c = 0.94, 0.90 and 0.97.

Obtain the forecast of standard deviation from volatility estimate model for the sample data to be used for estimation of Value at Risk (VaR)

Compute the daily Value at Risk (VaR) for Karachi Stock Index by using the Variance Covariance formula:
Value at Risk = a*s*P(t)*(-0.5)

1) **Simple Moving Average (SMA) Method:**

- Consider a time series of returns of a spot price or a constant maturity forward or future price $x_i, i = 1,...N,$
- Estimate of volatility:

$$\hat{\sigma} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(x_i - \mu)^2}$$

- Disadvantage: all the returns have the equal importance or weight.

Risk Metrics:

2) Historical Simulation Method:

The historical simulation, assuming that asset returns in the future will have the same distribution as they had in the past (historical market data),

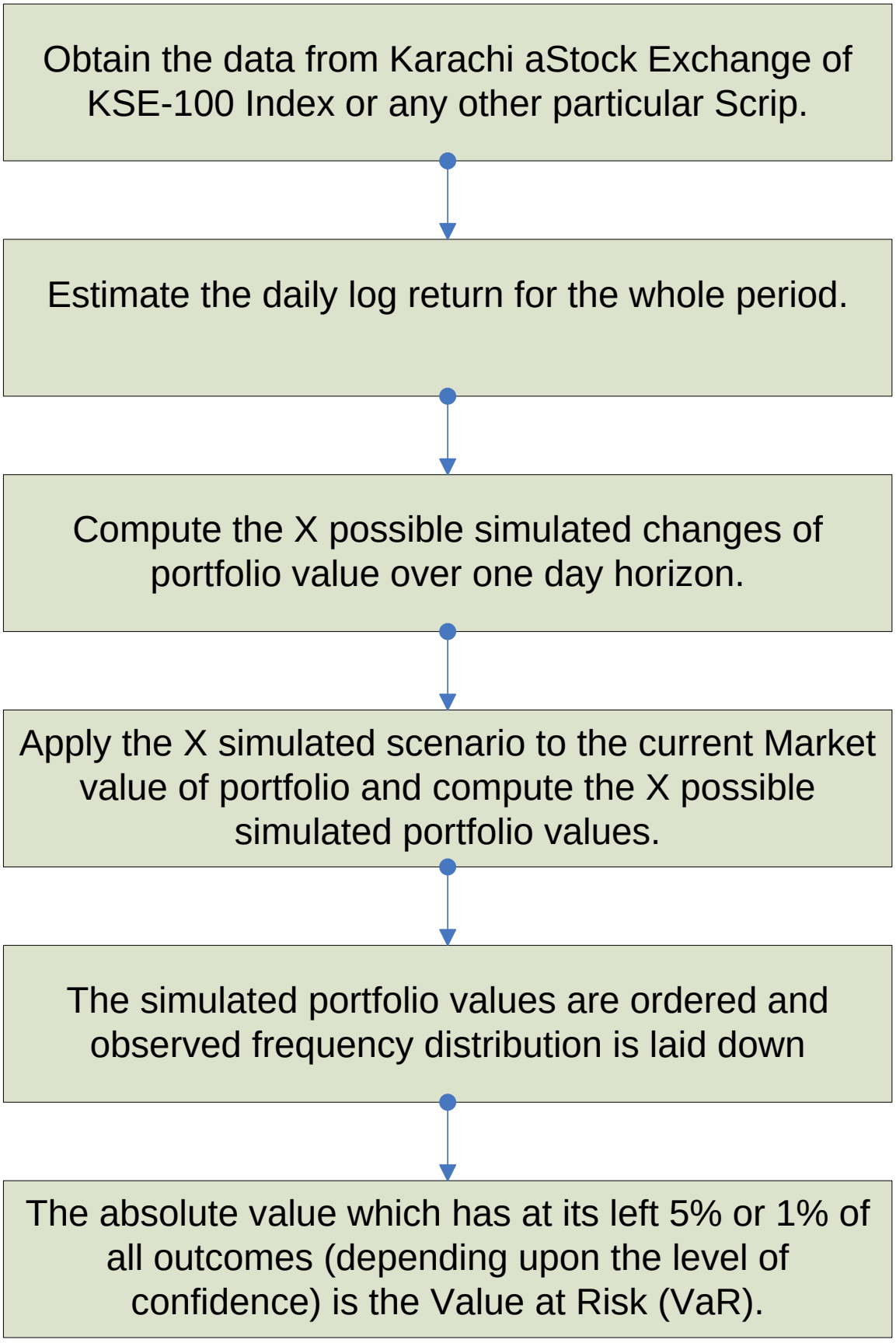
Assumption:

Uses past price movements to simulate what might happen today.

Historical simulation is the simplest and most transparent method of calculation. This involves running the current portfolio across a set of historical price changes to yield a distribution of changes in portfolio value, and computing a percentile (the VaR). The benefits of this method are its simplicity to implement, and the fact that it does not assume a normal distribution of asset returns.

In this way, the market factor is the stock value itself. Essentially the approach uses historical changes in market rates and prices to construct a distribution of potential future portfolio profits and losses, and then reads off the Value at Risk as the loss that is exceeded only 5% of the time. The main advantage of this approach over Variance Covariance methods lies in the fact that the approach obviates the need to calculate the correlations between market factors.

Even though the actual changes in rates and prices are used, the mark-to market profits and losses are hypothetical or simulated (hence the name for the approach) because the current portfolio was not held on each of the last *X* periods.


Obtain the data from Karachi aStock Exchange of KSE-100 Index or any other particular Scrip.
Estimate the daily log return for the whole period.
Compute the X possible simulated changes of portfolio value over one day horizon.
Apply the X simulated scenario to the current Market value of portfolio and compute the X possible simulated portfolio values.
The simulated portfolio values are ordered and observed frequency distribution is laid down
The absolute value which has at its left 5% or 1% of all outcomes (depending upon the level of confidence) is the Value at Risk (VaR).

4) Risk metrics exponentially weighted moving average (EWMA) model:

Assumptions:

There are generally two problems must try to solve when estimating volatility.

- They must keep the sample frame as wide as possible; the wider the window, the greater the number of Variables and so the more accurate the result. If the frame is kept narrow then the risk of sampling error is greater.
- They must recognize that more recent data bound to have a more important influence on future volatility than past data. This is because volatility tends to happen in clusters. If there is a stock market crash on, then volatility will remain high for the next two weeks of perhaps the next two months. After a while, volatility tends to return to sensible levels.

Formula:

One of the most popular volatility models in risk management framework is the Risk metrics model of JP Morgan® (1995), Give older returns exponentially less weight which is the following form:

$$\sigma_t^2 = \lambda\sigma_{t-1}^2 + (1-\lambda)(y_{t-1} - \hat{\mu}_t)^2,$$

Estimate of volatility:

$$\hat{\sigma} = \sqrt{\frac{1}{\sum_{i=1}^{N}\lambda^{i-1}}\sum_{i=1}^{N}\lambda^{i-1}(x_i-\mu)^2}$$

– Since $\sum_{i=1}^{N}\lambda^{i-1} = \frac{1-\lambda^N}{1-\lambda} \cong \frac{1}{1-\lambda}$, the formula for EWMA is $\hat{\sigma} = \sqrt{(1-\lambda)\sum_{i=1}^{N}\lambda^{i-1}(x_i-\mu)^2}$

The parameter $\lambda\ (0 < \lambda < 1)$ is called decay factor

Where

λ is exponentially declining factor,

Here λ reduces the emphasis that is placed on distant data and places more emphasis on recent data. The Risk metrics methodology assumes a fixed constant= 0.94 which substantially reduces the

volatility computations. This study considers additional values for, 0.90 and 0.97.

In trying to reach better precision on the forecast of future volatility, the idea of adjusting the time series method to forecast future volatility. The idea behind exponential weighting is to apportion weights to data contained in the moving average.

Where we use a time series to estimate volatility, we must give more weighting to recent events as opposed to older events. Two of the more popular methods of estimating volatility are EWMA and GARCH (generalized autoregressive conditional heteroscedasticity). While GARCH is more accurate, it is fairly difficult to implement in practice. EWMA is more practical and can every often achieve the accuracy of GARCH.

4. Monte Carlo simulation:

Monte Carlo simulation is conceptually simple, but is generally computationally more intensive than the methods described above. The generic Monte Carlo VaR calculation goes as follows:

- Decide on N, the number of iterations (random numbers) to perform.

- For each iteration:
    - Generate a random scenario of market moves using some market model (using normal probability model).
    - Use the spreadsheet function for normal distribution.
- Sort the resulting loss to give us the simulated loss distribution for the portfolio.
- VaR at a particular confidence level is calculated using the percentile function. For example, if we computed 5000 simulations, our estimate of the 95% percentile would correspond to the 250th largest loss, i.e. (1 - 0.95) * 5000.
- Note that we can compute an error term associated with our estimate of VaR and this error will decrease as the number of iterations increases.

Monte Carlo simulation is generally used to compute VaR for portfolios containing securities with non-linear returns (e.g. options) since the computational effort required is non-trivial. Note that for portfolios without these complicated securities, such as a portfolio of stocks, the Variance covariance method is perfectly suitable and should probably be used instead. Also note that MC VaR is subject to model risk if the market model is not correct.

Limitations of VaR:

While Value at Risk has acquired a strong following in the risk management community, there is reason to be skeptical of both its accuracy as a risk management tool and its use in decision making. There are many dimensions on which researcher have taken issue with VaR and we will categorize the criticism into those dimensions.

VaR can be wrong:

There is no precise measure of Value at Risk, and each measure comes with its own limitations. The end-result is that the Value at Risk that we compute for an asset, portfolio or a firm can be wrong, and sometimes, the errors can be large enough to make VaR a misleading measure of risk exposure. The reasons for the errors can vary across firms and for different measures and include the following.

a. **Return distributions**: Every VaR measure makes assumptions about return distributions, which, if violated, result in incorrect estimates of the Value at Risk. With normal estimates of VaR, we are assuming that the multivariate return distribution is the normal distribution, since the Value at Risk is based entirely on the standard deviation in returns. With Monte Carlo simulations, we get

more freedom to specify different types of return distributions, but we can still be wrong when we make those judgments.

Finally, with historical simulations, we are assuming that the historical return distribution (based upon past data) is representative of the distribution of returns looking forward. There is substantial evidence that returns are not normally distributed and that not only are outliers more common in reality but that they are much larger than expected, given the normal distribution.

b. **History may not a good predictor**: All measures of Value at Risk use historical data to some degree or the other. In the variance-covariance method, historical data is used to compute the variance-covariance matrix that is the basis for the computation of VaR. In historical simulations, the VaR is entirely based upon the historical data with the likelihood of value losses computed from the time series of returns.

In Monte Carlo simulations, the distributions don't have to be based upon historical data but it is difficult to see how else they can be derived. In short, any Value at Risk measure will be a function of the time period over which the historical data is collected. If that time period was a relatively stable one, the computed Value at Risk will be a low number and will understate the risk looking forward.

Conversely, if the time period examined was volatile, the Value at Risk will be set too high.

Back Testing:

Whatever the method used for estimating VaR, an important reality check is back testing. It involves testing how well the VaR estimates would have performed in the past. Suppose that we are calculating a one day 99% VaR back testing would involve looking at how often the loss in a day exceeded the one day 99% VaR that would have been calculated for that. If this happened on about 1% of the days, we can feel reasonably comfortable with the methodology for calculating VaR. If it happens on, say, 7% of the days, the methodology is suspect.

IMPLEMENTATION OF VaR ON EQUITY PRICES:

In order to implement the above described methodologies, the equity price of different scrip has used in the up forth section. In this section, VaR will be measured by using the above mentioned techniques and will be compared to each other to get the understanding of most reliable results.

For implementing, the equity prices of Muslim Commercial Bank (MCB), Fauji Fertilizer, Adamjee Insurance and Oil and Gas Development Company (OGDC), from July 2nd, 2007 to June 27th, 2008, have been selected as a portfolio.

Date	Adamjee Insurance	MCB	Fauji Fertilizer	OGDC
3-Jul-07	324.50	364.90	122.90	121.90
4-Jul-07	316.00	361.00	122.75	123.10
5-Jul-07	318.60	358.70	123.75	122.60
6-Jul-07	311.00	353.50	127.40	123.65
9-Jul-07	308.00	350.00	127.35	122.75
10-Jul-07	317.05	354.25	127.70	122.25
11-Jul-07	308.50	352.40	125.20	122.00
12-Jul-07	305.50	348.00	126.10	122.00
13-Jul-07	312.90	350.65	127.10	123.10
-	-	-	-	-
-	-	-	-	-

-	-	-	-	-
17-Jun-08	276.40	291.00	136.25	132.60
18-Jun-08	279.25	277.50	140.01	130.40
19-Jun-08	280.50	263.63	137.00	131.65
20-Jun-08	266.48	269.00	136.10	131.30
24-Jun-08	253.16	281.10	135.00	131.50
25-Jun-08	264.56	309.21	138.50	130.00
26-Jun-08	279.00	333.00	137.12	129.30
27-Jun-08	276.21	329.67	135.00	128.53

Their corresponding holdings have been used as weightage,

	Holdings	**Weightage %**
Adamjee Insurance	3,000,000	2.52%
MCB	20,000,000	16.82%
Fauji Fertilizer	9,869,485	8.30%
OGDC	86,018,568	72.35%
Total	118,888,053	100%

Furthermore, the log returns of series have been drawn to get the knowledge of past performance of the above mentioned scrip, same as the statistics of these returns will show us that how well these behaved.

LOG RETURN SERIES:

Period	Adamjee Insurance	MCB	Fauji Fertilizer	OGDC
1	-2.65%	-1.07%	-0.12%	0.98%
2	0.82%	-0.64%	0.81%	-0.41%
3	-2.41%	-1.46%	2.91%	0.85%
4	-0.97%	-1.00%	-0.04%	-0.73%
5	2.90%	1.21%	0.27%	-0.41%
-	-	-	-	-
-	-	-	-	-
-	-	-	-	-
224	-5.13%	2.02%	-0.66%	-0.27%
225	-5.13%	4.40%	-0.81%	0.15%
226	4.40%	9.53%	2.56%	-1.15%
227	5.31%	7.41%	-1.00%	-0.54%
228	-1.01%	-1.01%	-1.56%	-0.60%

STATISTICS OF LOG RETURN SERIES:

MIN	-5.13%	-5.13%	-5.12%	-5.13%
MAX	5.77%	9.53%	4.88%	4.87%
AVERAGE	-0.07%	-0.04%	0.04%	0.02%
St. Dev.	2.94%	2.76%	1.65%	1.51%

The above statistics exhibits, the average performance in the daily equity return has been best of Fauji Fertilizer among all the scrip but as far as the consistency is concerned, OGDC performed outstanding through out the past up and down scenario.

VALUE AT RISK (VaR) MEASUREMENT:

Our next step to estimate the VaR of an assumed portfolio to analyze and see how well it will perform in future.

STANDARD VaR:

This is the simplest method through which the VaR is estimated. In this methodology, the volatility is estimated by carrying out the standard deviations of each scrip and a portfolio.

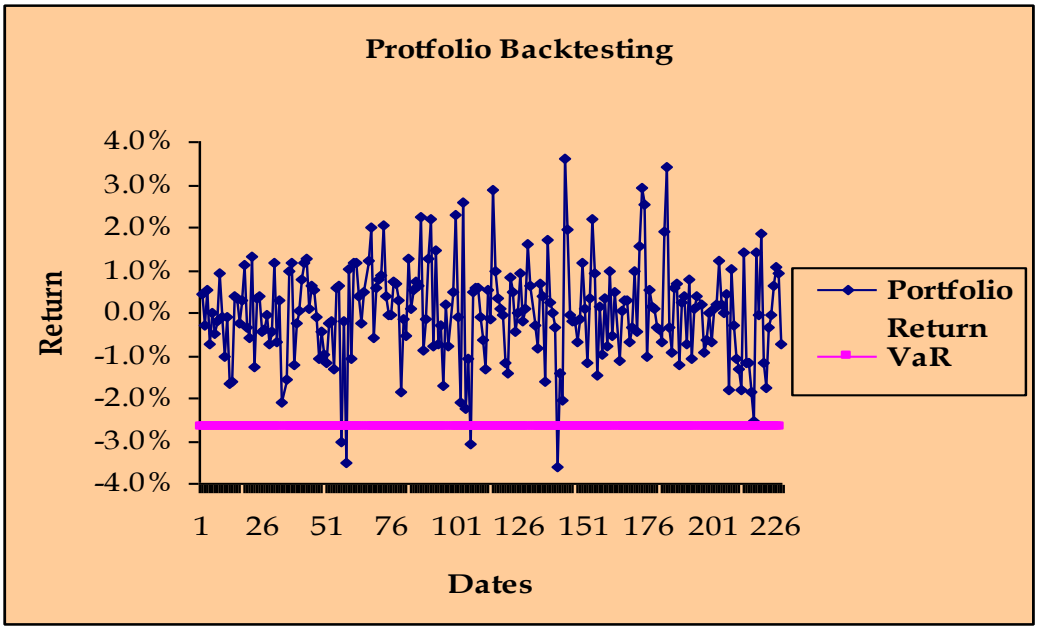
	Volatility %	VaR %	Weightage	VaR (Rs.)	VaR %
Adamjee Insurance	2.94%	6.83%	3,000,000	204,927	6.83%
MCB	2.76%	6.43%	20,000,000	1,285,462	6.43%
Fauji Fertilizer	1.65%	3.84%	9,869,485	379,106	3.84%
OGDC	1.51%	3.52%	86,018,568	3,030,278	3.52%

The above table shows the VaR of individual scrip which exhibits the maximum loss in one day horizon at about 99% confidence level can not be exceeded than the above mentioned limits in Rs.

Portfolio		
Vol	VaR	VaR %
1,370,571	**3,188,424**	**2.68%**

Similarly, the above mentioned table showing the over all loss in a portfolio instead of each scrip and it can not be exceeded, under the same condition, than 2.68%.

Back Testing Through Standard VaR:



WEIGHTED VaR:

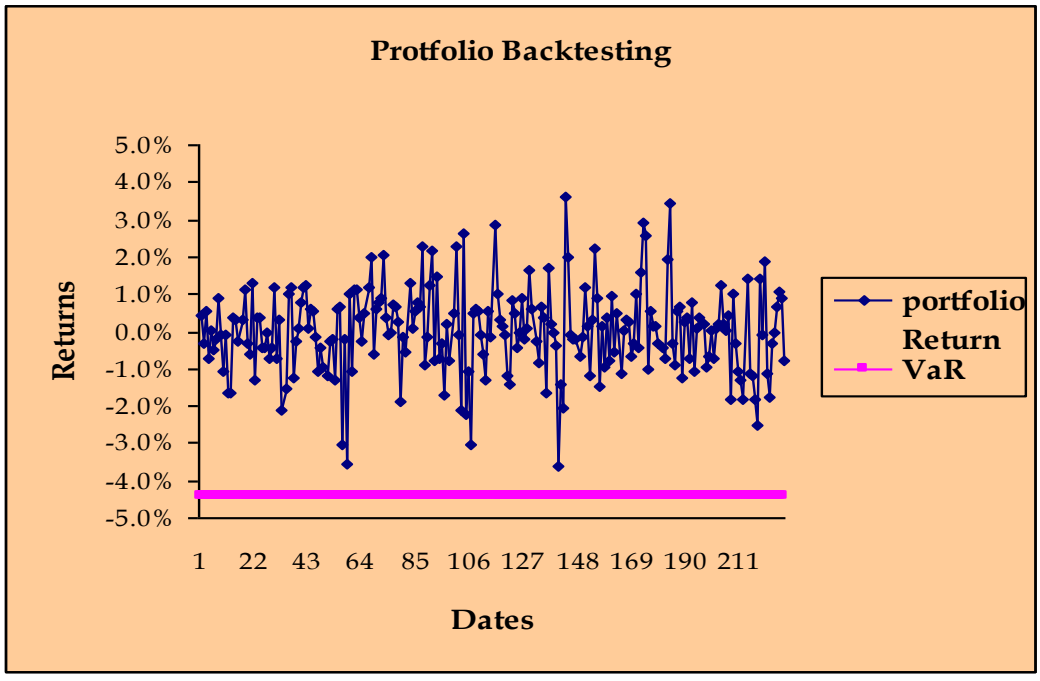
This methodology is little different and could be more accurate than the standard VaR. In this technique the total loss distributed among the scrip according to the weights. The variable, in which volatility is low, will be given more weights.

	Vol %	VaR %	Exposure	Weightage	Wt. VaR
Adamjee Insurance	3.73%	8.67%	3,000,000	2.52%	0.22%
MCB	4.59%	10.68%	20,000,000	16.82%	1.80%
Fauji Fertilizer	2.48%	5.77%	9,869,485	8.30%	0.48%
OGDC	1.13%	2.64%	86,018,568	72.35%	1.91%

Portfolio VaR	VaR (Rs.)	Total Investment
4.40%	5,235,291	118,888,053

Through this methodology, the risk amount has little increased, which indicates that the VaR could be varying with different methodology.

Back Testing Through Weighted VaR:



VARIANCE-COVARIANCE VaR:

In this methodology for VaR estimation, the volatility is measured by exponentially Weighted Moving Average (EWMA) by keeping the decay factor λ=0.94 for daily observations, which is more precise technique to evaluate volatility of a variable by using past observation or return series.

Variance-Covariance Matrix				
	Adamjee Insurance	MCB	Fauji Fertilizer	OGDC
Adamjee Insurance	3.7285%	0.0003%	0.0001%	0.0000%
MCB	0.0003%	4.5900%	0.0001%	0.0000%
Fauji Fertilizer	0.0001%	0.0001%	2.4792%	0.0000%
OGDC	0.0000%	0.0000%	0.0000%	0.0129%

The diagonal elements in the above table shows the variances and off diagonals are covariance between scrip.

	Adamjee Insurance	MCB	Fauji Fertilizer	OGDC
Portfolio %(Weightage)	2.52%	16.82%	8.30%	72.35%
Expectd Return	-0.071%	-0.045%	0.041%	0.023%
Varaince Terms	0.002%	0.130%	0.017%	0.007%
Return Terms	-0.002%	-0.007%	0.003%	0.017%

Variance	0.156%		
Volatility Of Portfolio	3.951%	**VaR**	9.20%
Return of Portfolio	0.011%		

By using Variance-Covariance technique, the results are quite different from other above defined methodologies. Around 9.2% VaR is estimated which is high in case of one day horizon.

VOLATILITY ESTIMATION THROUGH GARCH (1, 1) MODEL:

This methodology is quite near to the actual circumstances; it has the capability of removing autocorrelation from the previous observations and used to estimate the stochastic volatility. Results are given below of estimating VaR by using this technique.

The GARCH (1, 1) model is,

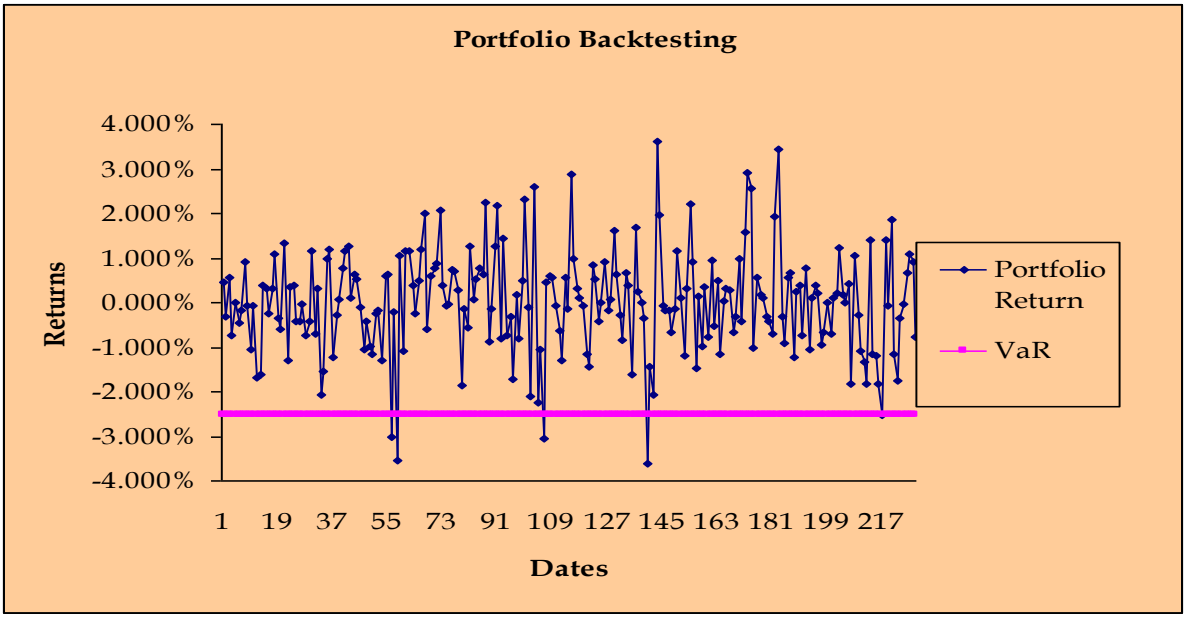
$$\sigma_n^2 = \gamma V_L + \alpha u_{n-1}^2 + \beta \sigma_{n-1}^2$$

$$\sigma_n^2 = 0.000057 + 0.177245 * u_{n-1}^2 + 0.393093 * \sigma_{n-1}^2$$

Hence, the following result is estimated by using the above equation of portfolio return series, which probably gives most precise result in all above defined techniques.

	GARCH VOL	VaR	Portfolio Volatility	Portfolio VaR
Adamjee Insurance	3.7164%	8.646%	1.086%	2.527%
MCB	3.742%	8.705%		
Fauji Fertilizer	1.5138%	3.522%		
OGDC	1.1119%	2.587%		

Back Testing Through GARCH (1, 1) VaR:

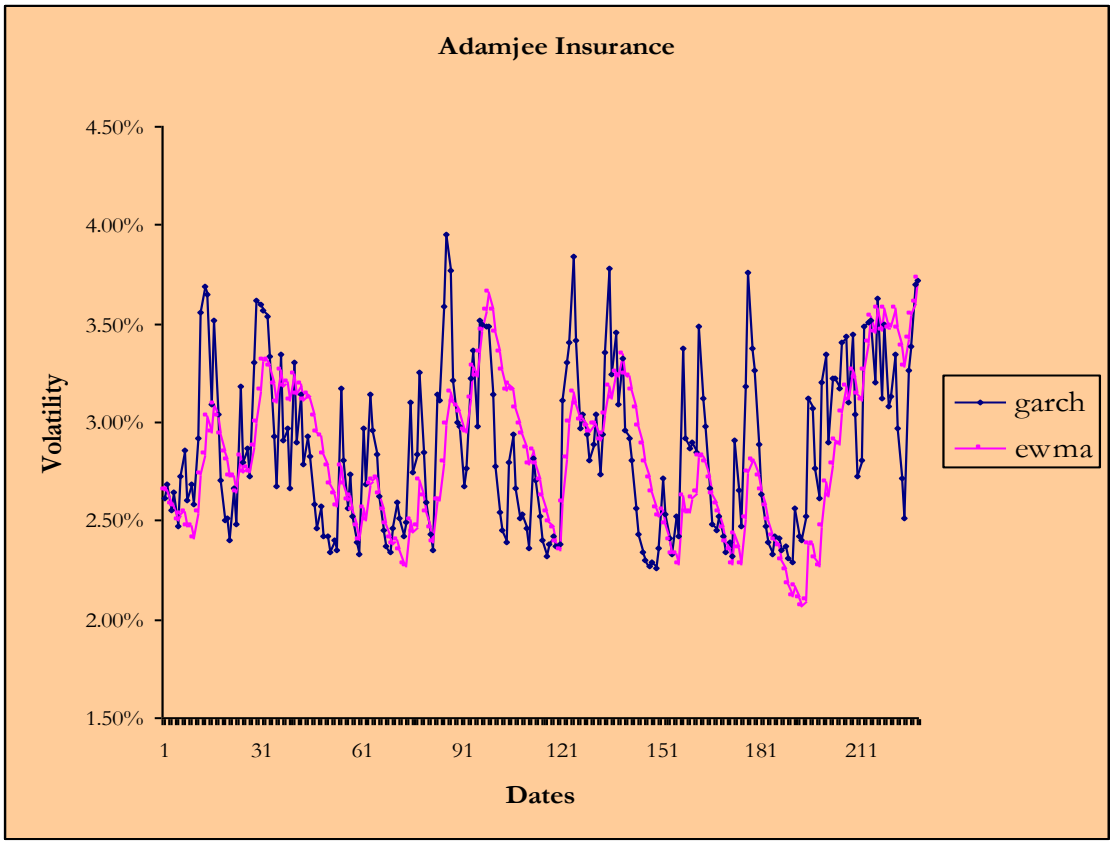


VOLATILITY CHECK:

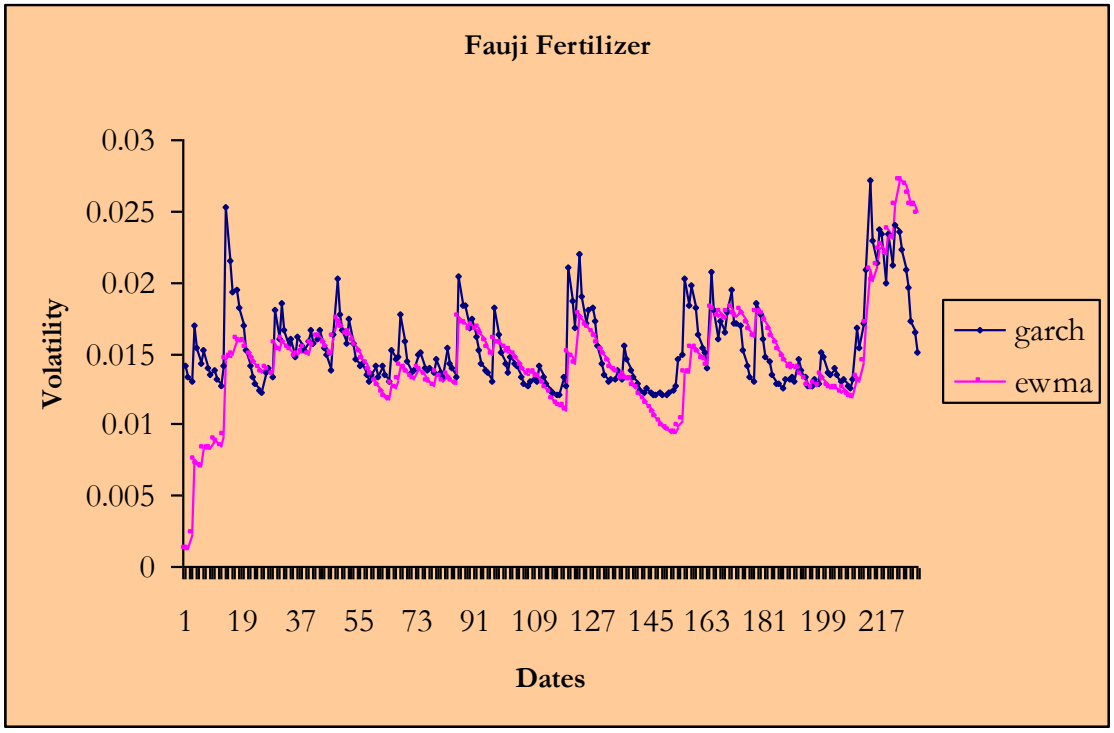
In this section, the volatility estimated with different methodology will be compared and analyzed. The most important thing is to evaluate is that there should be more precise estimation of variability

of market returns of variables which will be analyzed through volatility plots.

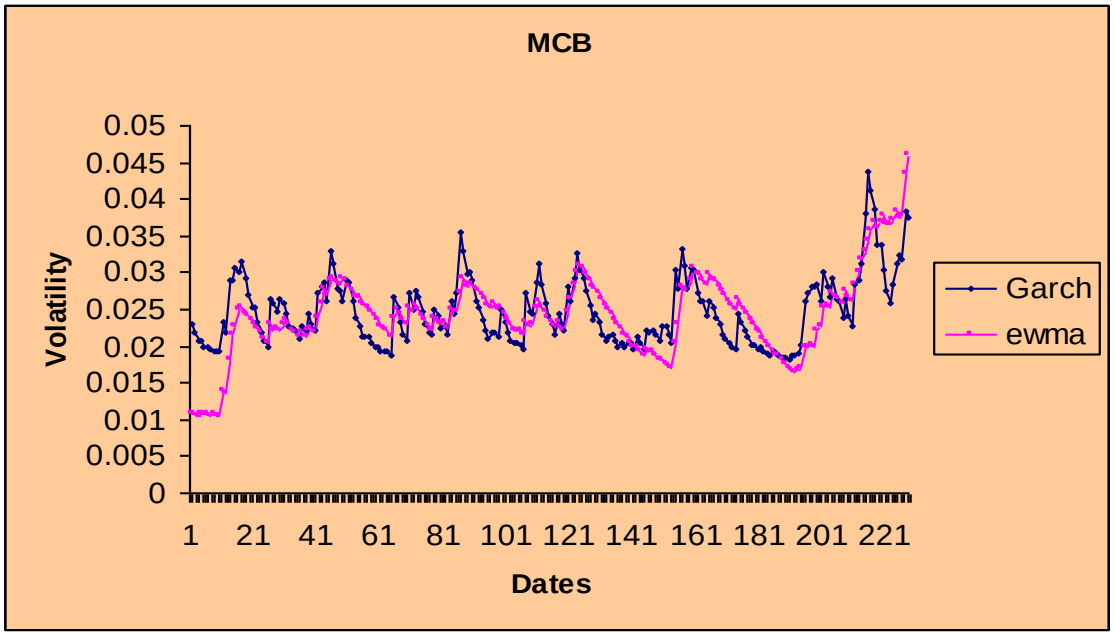
The following plot exhibits the variability by Exponentially Weighted Moving Average and GARCH (1, 1) models.

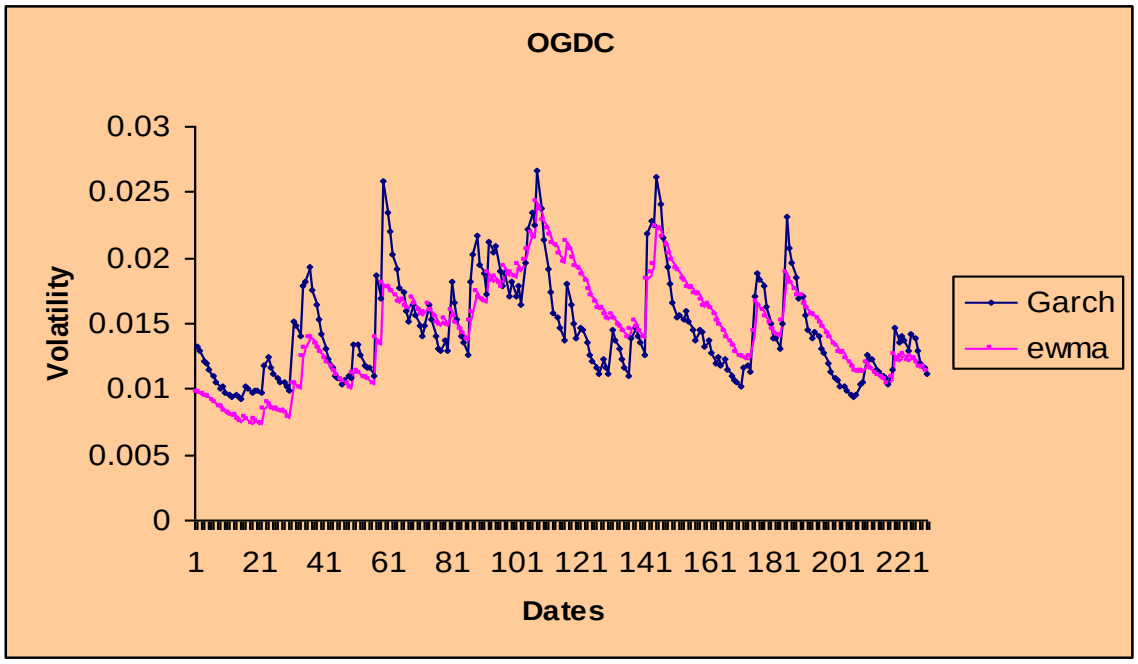


This above graph, Adamjee Insurance equity returns' Volatility, exhibits that the past ups and downs, in the return series, are estimated more precisely by GARCH model which may give more granularity in the model by this method. Similarly, this approach will be worthily for the remaining variables which is about to show below.



EWMA is also consistent technique to estimate volatility but sometimes it under estimates or over estimates the real scenarios, comparatively the GARCH technique is more convenient to use in practice.





These are some graphical representation of variables past performance and shows how the volatility behaves and captured their movements by using these two renowned methodologies.

Conclusion:

Value at Risk has developed as a risk assessment tool at Stock Exchanges, banks and other financial service firms in the last decade. Its usage in these firms has been driven by the failure of the risk tracking systems used until the early 1990s to detect dangerous risk taking on the part of traders and it offered a key benefit: a measure of capital at risk under extreme conditions in trading portfolios that could be updated on a regular basis.

While the notion of Value at Risk is simple- the maximum amount that you can lose on an investment over a particular period with a specified probability – there are three ways in which Value at Risk can be measured. In the first, we assume that the returns generated by exposure to multiple market risks are normally distributed. We use a variance-covariance matrix of all standardized instruments representing various market risks to estimate the standard deviation in portfolio returns and compute the Value at Risk from this standard deviation. In the second approach, we run a portfolio through historical data – a historical simulation – and estimate the probability that the losses exceed specified values. In the third approach, we assume return distributions for each of the individual market risks and run Monte Carlo simulations to arrive at the Value at Risk. Each measure comes with its own pluses and minuses: the Variance-covariance approach is simple to implement but the normality assumption can be tough to sustain, historical simulations assume that the past time periods used are representative of the future and Monte Carlo simulations are time and computation intensive. All three yield Value at Risk measures that are estimates and subject to judgment.

We understand why Value at Risk is a popular risk assessment tool in financial service firms, where assets are primarily marketable

securities; there is limited capital at play and a regulatory overlay that emphasizes short term exposure to extreme risks. We are hard pressed to see why Value at Risk is of particular use to non-financial service firms, unless they are highly levered and risk default if cash flows or value fall below a pre-specified level. Even in those cases, it would seem to us to be more prudent to use all of the information in the probability distribution rather than a small slice of it.

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